

Digital Signal Processing

**Part I - Digital Signals:
processing in computers and doing math**

Topic I.1 - Signal definitions and digitization

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www.lasse.ufpa.br & www.ppgee.ufpa.br

Definition of signal and some examples

General definition: anything that carries information

Image



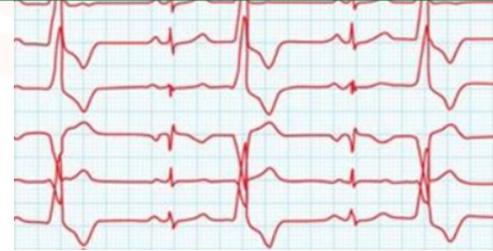
Monochromatic images are 2D signals, information are the pixel values, there is no time evolution, example: 3D matrix (tensor) $480 \times 640 \times 3$ (RGB)

Video



Sequence of images over time, example: 4D tensor $500 \times 480 \times 640 \times 3$ representing 500 frames $\mathbf{X}[n]$

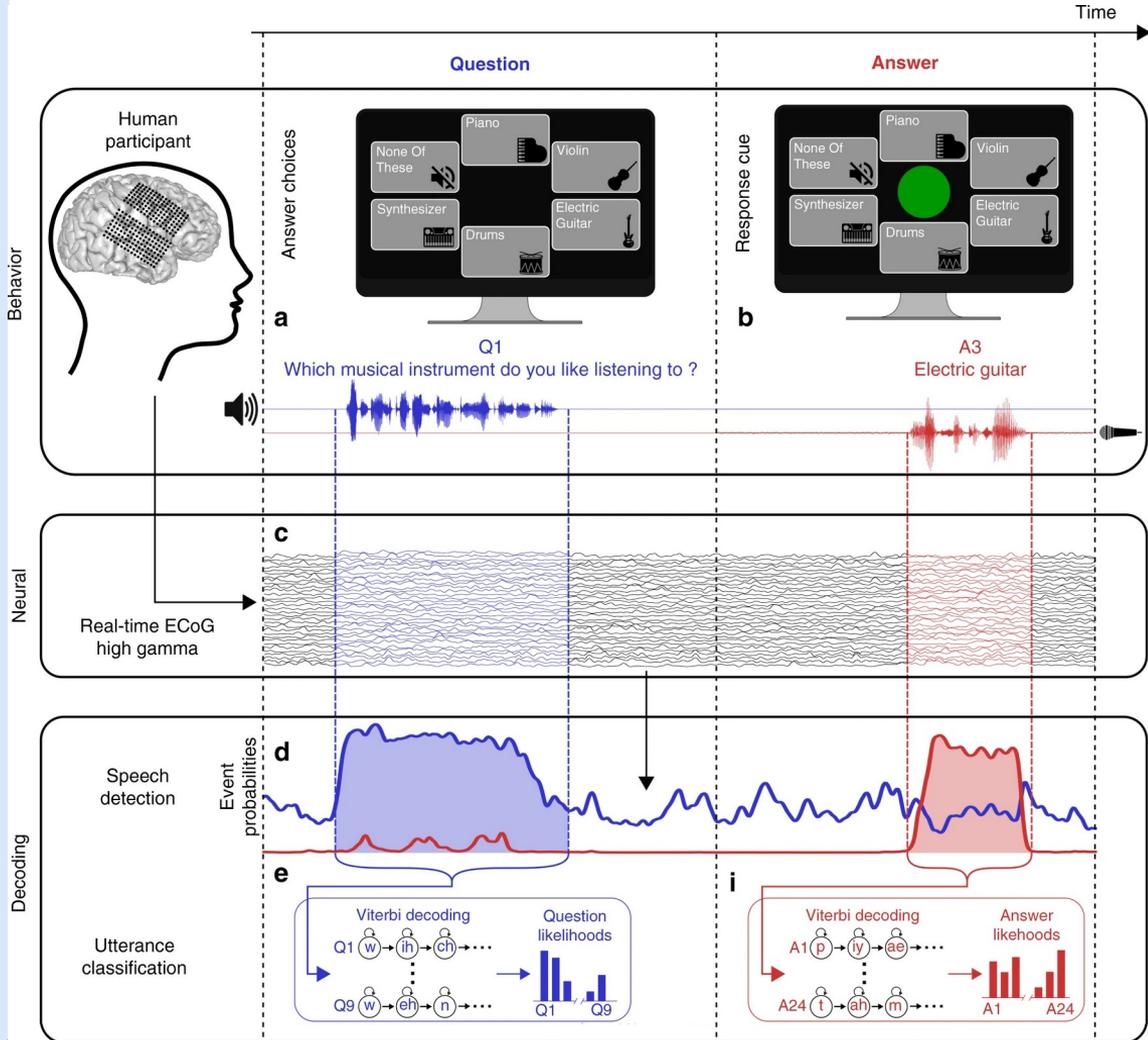
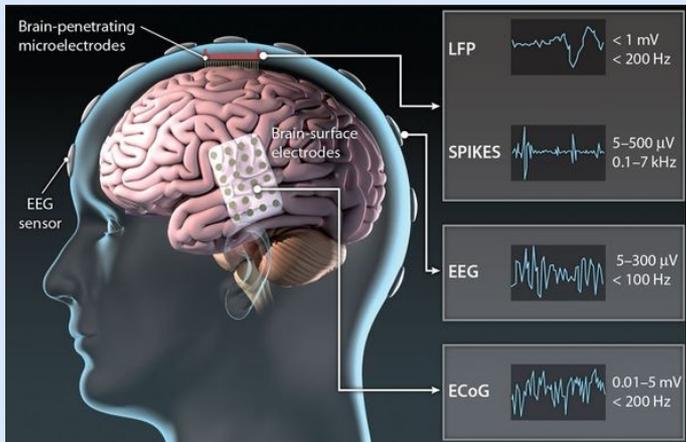
Electrocardiogram (ECG)



Multichannel ECG: Multivariate signal, information are amplitudes, example: 6 channels, with amplitudes in vector of dimension 6 $\mathbf{x}(t)$

Example of modern digital signal processing: decode brain signals

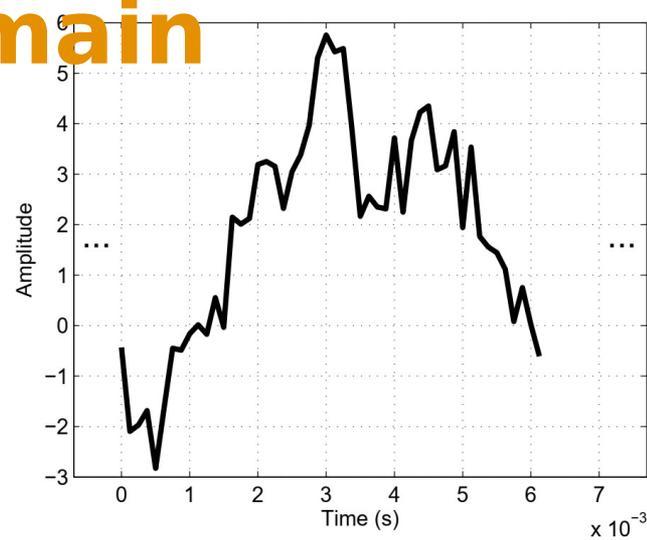
Electrocorticography (ECoG) vs electroencephalography (EEG)



Real-valued 1D time-domain signals

For concreteness:
Single dimension
Information is real

Time: continuous or discrete?



Types of signals

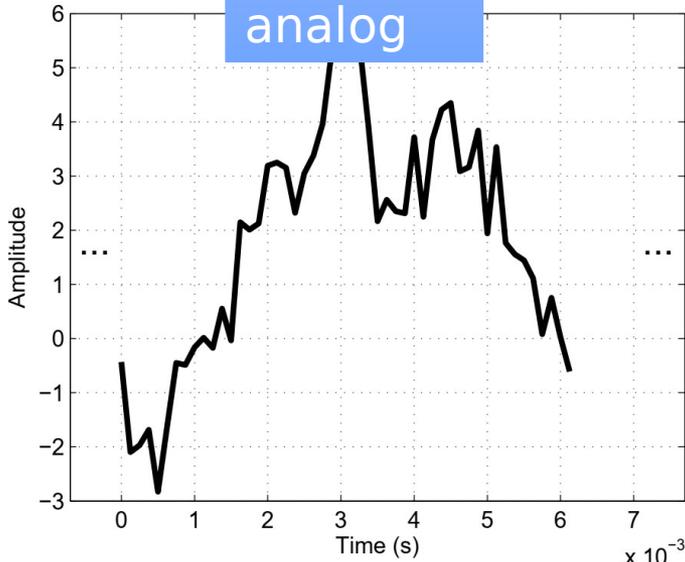
- If time evolution is represented by a real-valued variable $t \in \mathbb{R}$, the function $x(t)$ is called a *continuous-time signal*.
- If the progress over time is represented by an integer index $n \in \mathbb{Z}$, the sequence $x[n]$ is called a *discrete-time signal*.

Analog versus digital signals

Digital signal: discrete in both time and amplitude

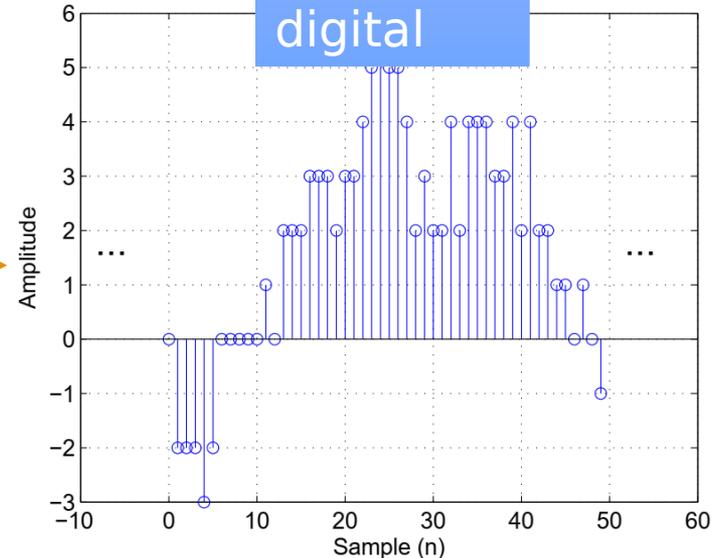
Analog signal: continuous in both (no quantization)

$x(t)$
analog



Amplitude quantized to 10 values in $\{-3, -4, \dots, 5, 6\}$ that could be represented with 4 bits

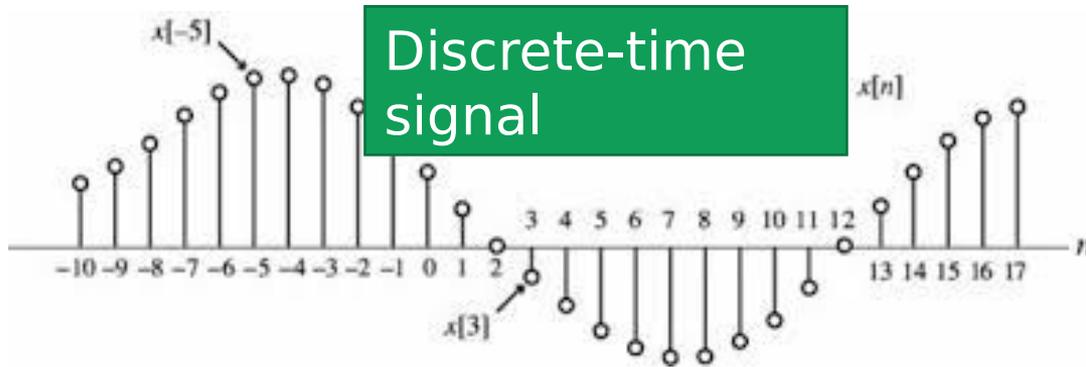
$x_q[n]$
digital



Four signal categories and notation



Table 1.1	Continuous-time, $t \in \mathbb{R}$	Discrete-time, $n \in \mathbb{Z}$
Quantized	$x_q(t)$	$x_q[n]$ (digital)
Not quantized	$x(t)$ (analog)	$x[n]$



Aldebaro Klautau

When doing math, we will be using **discrete-time** signals instead of digital signals

- The reason is that “quantization” is a non-linear operation that complicates incorporating, e.g., Fourier transforms and other operations

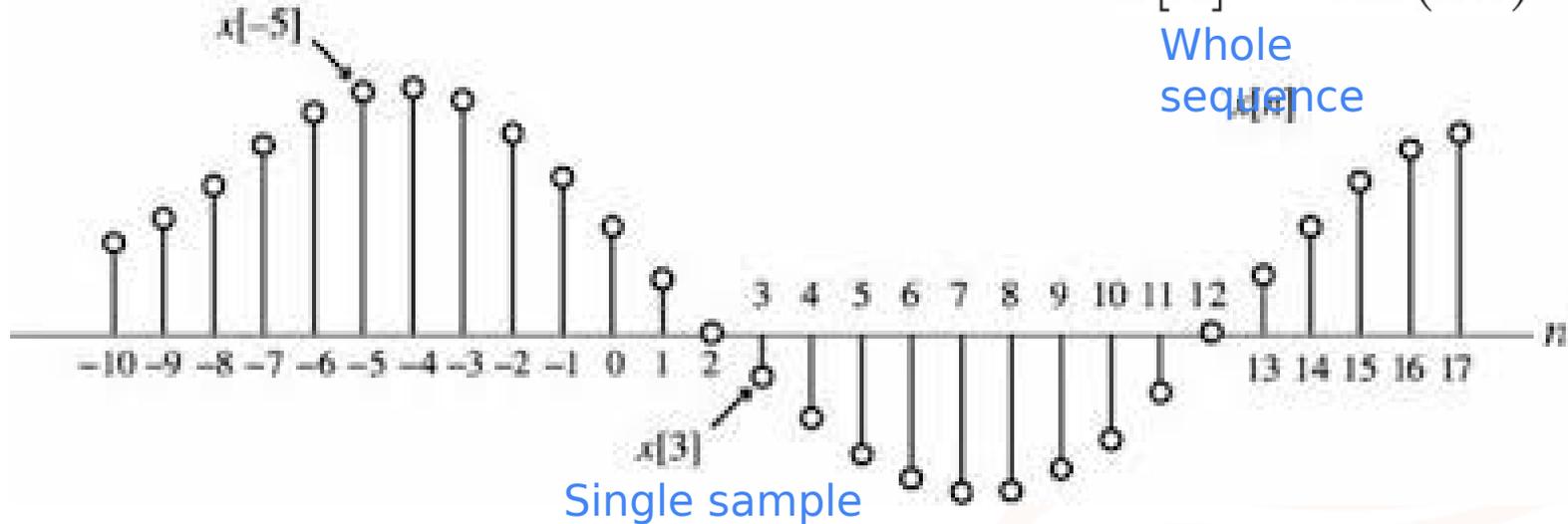
Beware: ambiguous notation!

The same notation $x[n]$ is used for both the whole sequence and for a specific sample at time n

Single sample

$$x[n] = \cos(3n)$$

Whole sequence



Notation: $x[n_0]$ (sample at time n_0) or $x[n]$ (sample

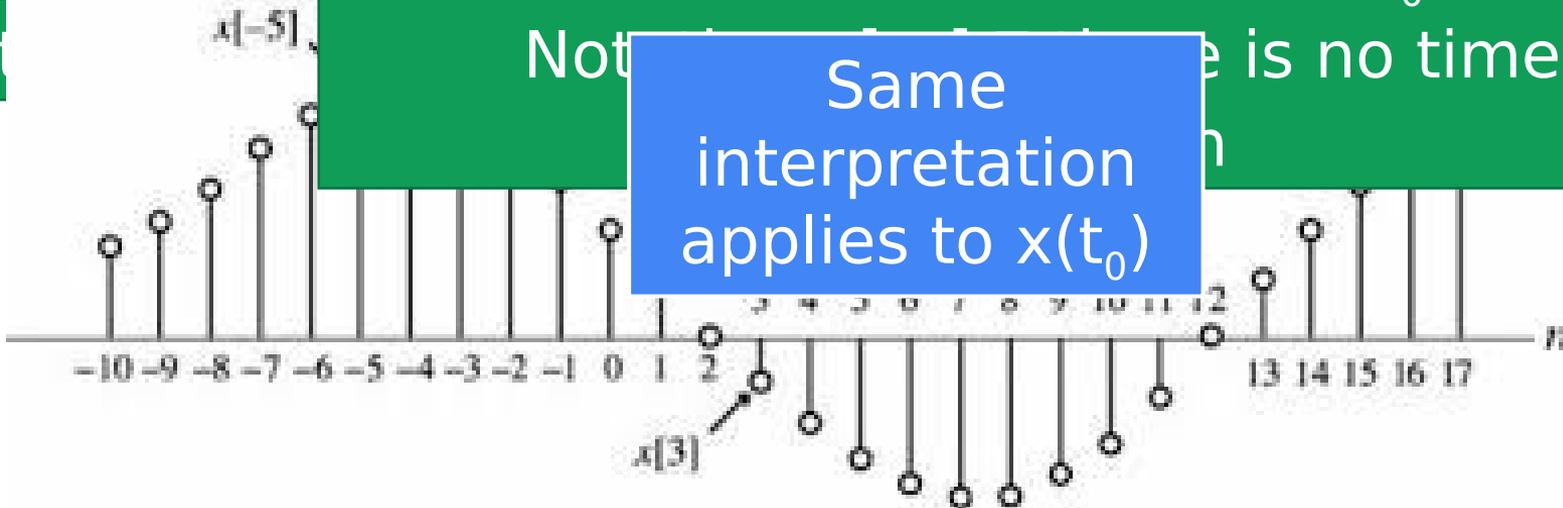
Notation (Our interpretation to its ambiguity)

We use n_0 or t_0 to denote a specific time instant

$x[n_0]$ is a sample at the given time n_0 , but we do not assume we can identify this time instant. Example: when $x[3] = -2$, we interpret $x[3]$ as the number -2. We do not know that -2 is associated with $n_0=3$.

Notation $x(t_0)$ is no time

Same interpretation applies to $x(t_0)$



Convention: assume signal $x(t)$ was obtained over a resistor with resistance = 1Ω

Advantage: we can get additional insight when interpreting results and expressions using units such as Watts

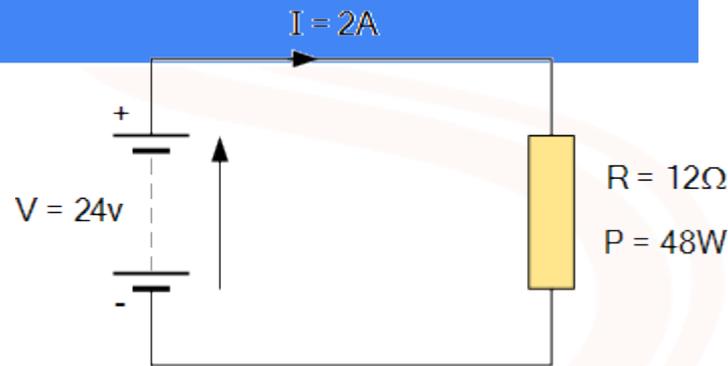
Remind simple circuit in which power is $P = V \cdot I$
 $= I^2 R$ Watts.

If $R = 1 \Omega$, then: $P = V^2 = I^2$

In DSP, we often assume $R = 1 \Omega$, such that the instantaneous power is

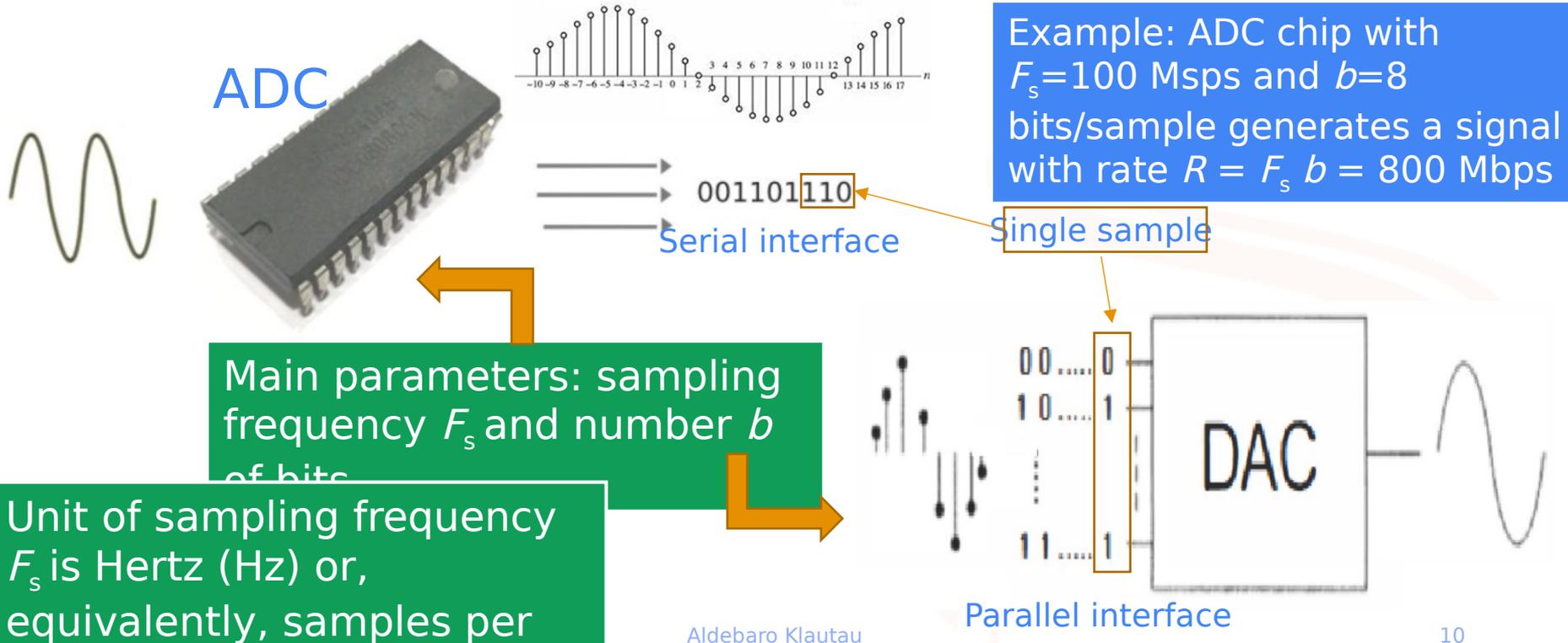
$$p(t) = x^2(t)$$

and we do not worry if $x(t)$ is in volt or ampere



Obs: If the resistance is not 1Ω , one can use “impedance scaling”

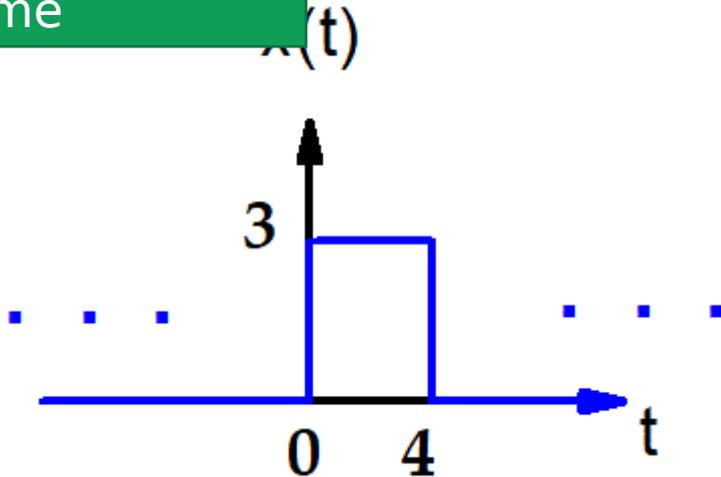
Analog-to-digital chip (ADC) Digital-to-analog chip (DAC)



Quiz 1

1.1. Both continuous-time $x(t)$ and the discrete-time $x[n]$ signals consist of pulses with a limited duration (finite support). Their amplitudes are equal to three for $0 \leq t \leq 4$ and $0 \leq n \leq 4$, respectively, and zero otherwise. a) What are the values of $x(t)$ when $t = 1.5$ and $t = 6$? b) What are the values of $x[n]$ when $n = 1.5$ and $n = 6$?

Continuous-time

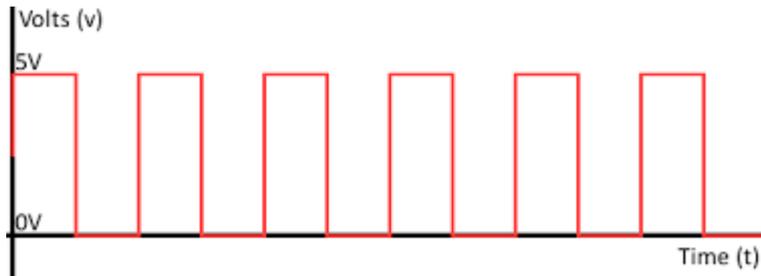


- How do we draw the graph of the discrete-time signal $x[n]$?
 - How many non-zero samples does $x[n]$ have in this case?

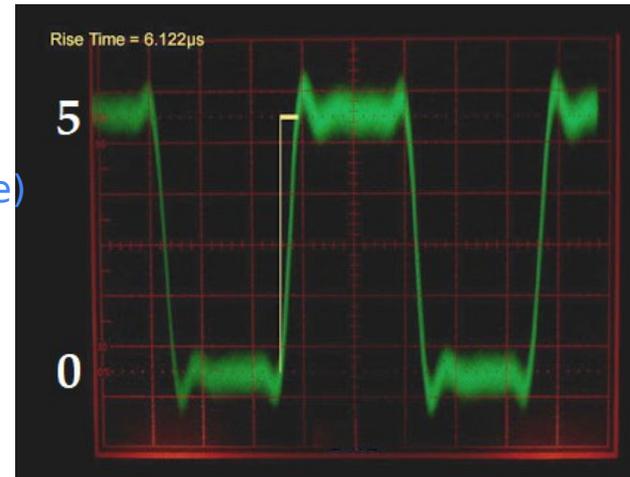
Quiz 2

1.2. Note the several distinct meanings of the word *digital* depending on the context: an FPGA chip implements *digital* logic, the signal analyzer has both analog and *digital* inputs, etc. According to the definitions adopted in this text, is the waveform at one of a microprocessor data bus pin a *digital* signal? Assume two cases for this waveform: a) only assumes two values: 0 and 5 V, and b) is composed by non-ideal pulses, with e. g. non-zero rise and fall times. If not digital, what are the classification types in these two cases according to Table 1.1)?

a) Idealized pulse train



b) Practical pulse train (from oscilloscope)



Quiz 3

1.2. An ADC uses $b = 8$ bits per sample and sampling frequency $F_s = 100$ Msps. a) What is the respective bit rate in bits per second? b) How many megabytes are needed to store 1 hour of a signal digitized with this ADC and stored in a binary “raw” format (without compression or a file header)?

Quiz 4

1.13. Get familiar with digitizing systems and boards, which are sometimes called DAQ (data acquisition) boards. Calculate the storage space and transfer rate for digitized signals (visit [[url1bww](#)] for extra information). a) Calculate the total space in megabytes (MB) for storing 30 minutes of a signal sampled at the maximum rate of the following data transfer technologies:

- PCI: 2133 Mbit/s (266.7 MB/s)
- Serial ATA (SATA-300): 3000 Mbit/s (375 MB/s)
- USB 2.0: 480 Mbit/s (60 MB/s)
- Serial RS-232 (max): 0.2304 Mbit/s (0.0288 MB/s)

b) Assume you need to use a 16-bits A/D process to achieve the desired SNR, what is the maximum sampling rate that needs to be supported for each interface above?

https://en.wikipedia.org/wiki/List_of_interface_bit_rates

Quiz 4 (continuation)

c) Describe in high-level a digitizer system to sustain a sampling rate of 40 MHz and store 3 hours of a signal. Choose the data transfer technology, total hard disk space, etc.). d) Considering you must use USB 2.0: what is the maximum sampling rate the system could achieve in this case? e) Evaluate a Signatec [[url1sig](http://www.signatec.com/)] waveform recording product and indicate what is the maximum throughput that Signatec offers (indicate sampling rate and number of bits per sample) for recording some hours of signal into a hard disk. Note that when operating at maximum sampling rate, most acquisition boards and digital oscilloscopes store the ADC samples in a limited amount of onboard RAM, which is typically capable of storing only few seconds of signal. The discussed recording system must take into account the data transfer from onboard RAM to hard disk.

<http://www.signatec.com/>

Quiz 5

1.12. Practice choosing commercial ADCs and DACs. Some companies of interest are Analog Devices, Maxim and Texas Instruments. Assume you have to choose chips for three projects with distinct requirements: 1) the fastest ADC and DAC with at least 16 bits per sample, 2) Low cost chips with 8 bits per sample to work with F_s up to 10 kHz and 3) high precision chips to work with F_s around 100 Hz in medical applications. In your comparison, indicate at least resolution, speed, price, power consumption, supply current and if the data bus is serial or parallel and inform the interface (e.g., SPI). Extra parameters you may include are full scale range (FSR), total harmonic distortion (THD), effective number of bits (ENOB) and offset error.

Check, for instance:

www.maximintegrated.com/en/products/analog/data-converters/analog-to-digital-converters/high-speed